

Synchronization of SA and AV Node Oscillators using PSO optimized RBF-based controllers and Comparison with SMC

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ABSTRACT: This paper studies the synchronization of SA and AV Node Oscillators using PSO optimized RBF-based controllers systems. High levels of control activities may excite unmodeled dynamics of a system. This changes the rules for controlling the system and achieving an acceptable control performance. In fact, the objective here is to reach a trade-off between tracking performance and parametric uncertainty. Two methods are proposed to synchronize the general forms of Van Der Pol (VDP) Model, and their performance. These methods use the radial basis function (RBF)- based neural controllers for this purpose. The first method uses a standard RBF neural controller. Particle swarm optimization (PSO) algorithm is used to derive and optimize the parameters of the RBF controller. In the second method, with the aim of increasing the robustness of the RBF controller, an error integral term is added to the equations of RBF neural network. For this method, the coefficients of the error integral component and the parameters of RBF neural network are also derived and optimized via PSO algorithm. For better comparison Simulation results show the effectiveness and superiority of the proposed methods in both performance in comparison with SMC controller.

Keywords: Synchronization, Van der Pol Model, SA and AV Node Oscillators, SMC.

INTRODUCTION

The present paper examines synchronizations for Van der Pol oscillatory systems. Synchronization problem has found many applications in laser, chemical reactors, secure communications, and biology. This paper deals with one such application in cardiac synchronization. This is particularly important as cardiovascular diseases are among the major causes of death worldwide. Disruption in the electrical function of the heart is a type of such diseases generally referred to as "cardiac arrhythmia". Thus, electrical conduction system of the heart can be modeled and used in preventing serious heart diseases. One practical way to investigate how a member of an organism works is to develop a model which accurately reflects the function of this part. Such a model may serve as a hypothesis for some physiological observation. For simulating how stimulation propagates over the heart tissue, it seems necessary to develop an accurate model of action potential of cells. For this purpose, Van der Pol model was used in the present study to examine synchronization of heart oscillators. The main goal of this study is to synchronize atrio-ventricular (AV) oscillator with sino-atrial oscillator based on a particular model and using different methods. We will also discuss how pacemakers, including SA node and AV node, can be resynchronized in cases where one is out of synch with the other (this is a major cause of arrhythmia).

2. An Overview of Cardiovascular Physiology

The heart will not be able to pump unless it receives an electrical excitation which originates pumping. Generation and transmission of electrical impulses depend on automaticity, excitability, conductivity, and contractibility of cardiac cells. Transmission of cardiac impulses creates depolarization-repolarization cycles in cardiac cells. When at rest, the cardiac cells are polarized, *i.e.* they show no sign of electrical activity. The cell membranes separate different concentrations of such ions as K^+ and Na^+ and create larger negative charges inside the cell. The phenomenon is known as resting membrane potential. As soon as an electrical excitation arrives, the ions are transported at either side of the cell membrane leading to action potential or depolarization. Once a cell is completely depolarized, it tries to return to its initial conditions or resting state. This process is referred to as repolarization. The electrical charges are reversed and return to the normal state. A typical depolarization-repolarization cycle consists of five phases (0 to 4) (Fig. 1 presents action potential curve and variations in voltage in these five phases):

Phase 0:

A cell receives an impulse from its adjacent cell and becomes depolarized.

Phase 1:

An initial immediate repolarization takes place.

Phase 2:

This slow repolarization step is also known as plateau phase. In Phase 1, Phase 2, and early in Phase 3, cardiac cells are at total inexcitability state. In this phase, not every stimulus with any intensity can result in cellular response.

Phase 3:

This phase is known as rapid repolarization. At this time, the cell returns to its initial state. At the last one-third of this phase, when the cell enters the relative excitability state, very strong excitations can depolarize it.

Phase 4:

This step is the resting state for action potential. By the end of the fourth phase, the cell is ready for next excitations. All these activities can be recorded on electrocardiogram (ECG).

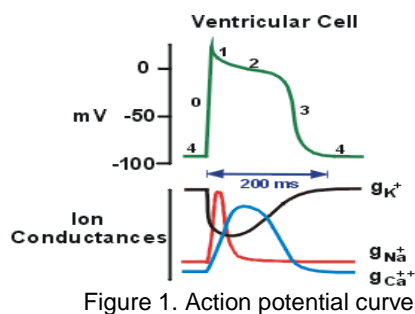


Figure 1. Action potential curve

2-1. Electromechanical Conduction mechanism of the Heart

Immediately after depolarization and repolarization, electrical impulses propagate along a pathway known as conduction system (Fig. 2). These impulses start traveling out of the SA node, through the atrium and Bachmann's bundle, and into the AV node. The impulses then travel through the bundle of His, left and right branches, and eventually into the Purkinje fibers. This conduction system is an electromechanical one. The electrical section orders the contraction of all cells and the mechanical section (cardiac muscles) implements these orders. Some diseases are caused by failure in these mechanical functions, while most diseases are the result of the malfunction of electrical system. Electrical conduction system of the heart can be thought of as a self-exciting pacemaker. This system is responsible for proper and synchronized contraction of cardiac muscles (2).

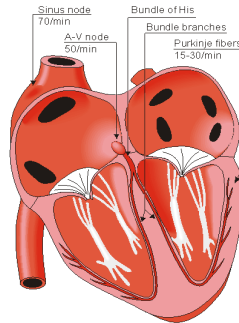


Figure 2. Pacemakers and impulses routes

3. Introduction to Synchronization

The word “synchronous” has its origin in the Greek word meaning “sharing the same time period”, and since its origin, the word has been used in everyday applications to denote agreement or dependency of the different processes in terms of time. Historically, analysis of synchronization of dynamic systems has received considerable attention as a very important subject in physics. The phenomenon dates back to the 17th century when Huygens patented two synchronized pendulum clocks with very weakly coupled oscillations.

In synchronization of oscillatory systems, two identical systems oscillate simultaneously. If one system is designated as master and another identical system is assigned as slave, when a proper control input is applied to the slave, the dynamic behavior of the two systems will become identical after a period of time. The slave, which often has to become synchronized with the master, is usually referred to as the response system or receiver while the master is sometimes called the drive or sender. The figure below, for example, shows synchronization of state trajectories of two systems after a few seconds.

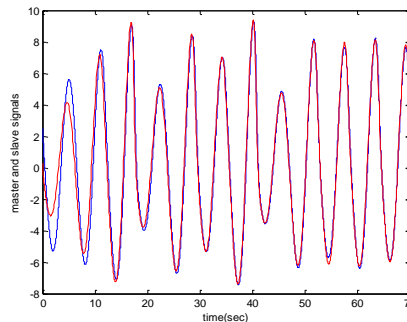


Figure 3. Synchronization

As mentioned earlier, the objective here is to synchronize the slave with the master. For this purpose, a nonlinear control system must be designed to receive the control signals from the master and control the slave.

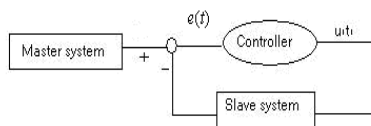


Figure 4. A schematic of master-slave combination

Fig. 4 presents a schematic of a master-slave system. Here, behavior of the slave is clearly controlled by the master. In addition, the slave may have conditions different from those of the master.

3-1- Van Der Pol (VDP) Model

The first attempts to explain oscillatory behavior of the heart cells was made in 1926 by Van der Pol [34]. Balthasar van der Pol was a German physicist and an electronic engineer. He was able to discover stable oscillations, now called limit cycle. Van der Pol was the first person to examine relaxation oscillations by studying an electrical

circuit which had self-entertained oscillations with the amplitude independent of initial conditions. The schematic of this circuit is shown in Fig. 5.

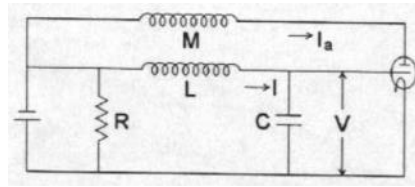


Figure 5. Van der Pol's circuit

The equations of the voltages and currents in this circuit are

$$I_a = CV'(\theta) \quad I_a = V - \frac{1}{3}V^3, \quad L \frac{dI}{d\theta} + RI + V = M \frac{dI_a}{d\theta} \tag{1}$$

where

$$c = \frac{M}{\sqrt{LC}} - R\sqrt{\frac{C}{L}}, \quad \theta = t\sqrt{LC} \quad V = x\sqrt{1 - R\frac{C}{M}} \tag{2}$$

By substitution x , t , and c from (2) into the current-voltage equations in (1), the following differential equation, known as Van der Pol's equation, is obtained:

$\frac{d^2x}{dt^2} + c(x^2 - 1)\frac{dx}{dt} + x = 0$ This circuit serves as an essential model for self-entertained oscillations in physics, electronic engineering, biology, neurology, and many other sciences. Since c is the control parameter in this equation, different periodic responses can be initiated by changing the value of c with large values of c resulting in relaxation oscillations.

For some important properties, Van der Pol nonlinear equations are used to model the oscillations in the heart. First, Van der Pol oscillator adjusts its natural frequency to the frequency of the input signal without changing the oscillation amplitude. This is critically important as the low-frequency slave oscillator has to adjust itself to the dominant high-frequency pacemaker of the heart.

Therefore, Van der Pol model used in this project to model the oscillators at SA and AV nodes. Each oscillator at SA node and AV node is modeled using Van der Pol differential equations. The interaction between the oscillators of the heart is modeled by the following Van der Pol equations. The coupling between these interacting oscillators is modeled as follows:

$$SA: \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -w_1^2 x_1 + c_1(\mu - x_1^2)x_2 + R_1(x_4 - x_2) \end{cases} \tag{3}$$

$$AV: \begin{cases} \dot{x}_3 = x_4 \\ \dot{x}_4 = -w_2^2 x_3 + c_2(\mu - x_3^2)x_4 + R_2(x_2 - x_4) + u \end{cases} \tag{4}$$

The first equation, which models SA oscillations, is the drive in the present synchronization problem while the second equation, for modeling AV oscillations, represents the response system.

4. Generating Action Potential by the Model

The rhythm of the heart is determined by a series of electric impulses (action potential) which travel throughout the heart. The figures below show action potentials for SA and AV cells obtained through.

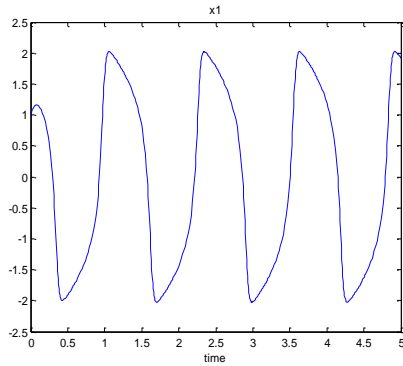


Figure 6. Simulating Van der Pol Model in MATLAB

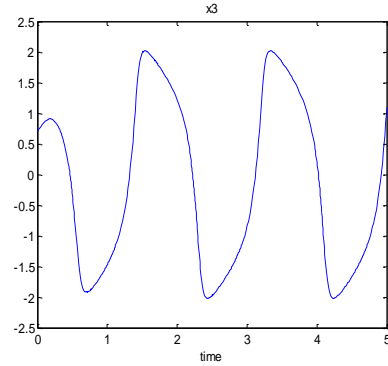


Figure 7. Action potential waveforms

These figures demonstrate the validity of the Van der Pol model used here in terms of how these waveforms match the actual forms generated in the heart.

5. Designing Sliding Mode Controller

Sliding mode control provides a robust controller for nonlinear dynamic systems, although it involves a discrete-time control factor which may lead to chattering in a neighborhood of the sliding surface. One common solution to reduce chattering is to introduce a boundary layer in the vicinity of the sliding surface [11]. This can improve closed-loop stability and resolve chattering, but a finite steady-state error will exist as a result of finite steady-state error gain in the control algorithm.

The proposed controller provides a desirable transient state as well as a robust performance. In addition, it combines PI control and SMC to prevent chattering. It will be demonstrated that the closed-loop system is globally stable in the sense of Lyapunov (i.s.L.) and that the output of the system can asymptotically track the reference input by modeling uncertainties and disturbances.

6. Problem Definition

A typical nonlinear SISO system of order n can be shown as

$$\begin{aligned} x^{(n)} &= f(\underline{x}, t) + g(\underline{x}, t)u + a(t) \\ y &= x \end{aligned} \tag{5}$$

where f and g are unknown nonlinear functions, and $\underline{x} = [x, \dot{x}, \dots, x^{(n-1)}]^T = [x_1, x_2, \dots, x_n]^T \in R^n$ is the state vector of the system, assumed to be measurable. $u \in R$ and $y \in R$ represent the system input and output, respectively and $d(t)$ is an unknown outer disturbance. It is assumed that $d(t)$ is bounded by the upper bound D ; i.e. $d(t) \leq D$. For the system (5) to be controllable, we must have $g(\underline{x}, t) \neq 0$. Therefore, we assume that $g(\underline{x}, t) > 0$. The control problem is to find the state x for tracking the desirable state x_d in the presence of uncertainties and outer disturbances with the following tracking error:

$$\underline{e} = \underline{x} - \underline{x}_d [e, \dot{e}, \dots, e^{(n-1)}]^T \in R^n \tag{6}$$

The sliding surface for the state error can be determined as

$$s(\underline{e}) = c_1 e + c_2 \dot{e} + \dots + c_{n-1} e^{(n-2)} + e^{(n-1)} = \underline{c}^T \underline{e} \tag{7}$$

where $\underline{c} = [c_1, c_2, \dots, c_{n-1}, 1]^T$ represents the coefficients of the Hurwitz polynomial $h(\lambda) = \lambda^{n-1} + c_{n-1}\lambda^{n-2} + \dots + c_1$. All roots lie in the left-hand plane and λ is the Laplace operator. If the initial condition $e(0)=0$ holds, in the tracking problem $\underline{x} = \underline{x}_d$, an error vector on the state space may be defined on the sliding surface $s(\underline{e})=0$ for all $t>0$. A sufficient condition for this, is to choose a control strategy as

$$\frac{1}{2} \frac{d}{dt} s^2(\underline{e}) \leq -\eta |s| \quad , \quad \eta \geq 0 \tag{8}$$

The state of the controlled system always moves toward the sliding surface. The sign of the control signal must switch based on the state trajectory and the sliding surface. Consider the nonlinear control problem shown in (5). If $f(\underline{x}, t)$ and $g(\underline{x}, t)$ are known, the SMC input u^* satisfies the sliding condition (8).

$$\tag{9}$$

$$u^* = \frac{1}{g(\underline{x}, t)} \left[- \sum_{i=1}^{n-1} c_i e^{(i)} - f(\underline{x}, t) + \dot{x}_d^{(n)} - \eta \text{sgn}(s) \right]$$

where

$$\text{sgn}(s) = \begin{cases} 1 & \text{for } s > 0 \\ 0 & \text{for } s = 0 \\ -1 & \text{for } s < 0 \end{cases} \quad (10)$$

A Lyapunov candidate function is defined as:

$$V_1 = \frac{1}{2} s^2(\underline{e}) \quad (11)$$

By differentiating (11) with respect to time, \dot{V} for the system trajectory is obtained:

$$\begin{aligned} \dot{V} &= s \cdot \dot{s} = s \cdot (c_1 \dot{e} + c_2 \ddot{e} + \dots + c_{n-1} e^{(n-1)} + x_d^{(n)} + \dot{x}_d^{(n)}) \\ &= s \cdot \left(\sum_{i=1}^{n-1} c_i e^{(i)} + f(\underline{x}, t) + g(\underline{x}, t)u + d(t) - \dot{x}_d^{(n)} \right) \leq -\eta |s| \end{aligned} \quad (12)$$

Therefore, the SMC input u^* satisfies the sliding condition (8). Clearly, to satisfy this condition, an impulsive control condition must be added in the form of $u^* = u_{eq} - u_{sw}$

where

$$u_{eq} = g(\underline{x}, t)^{-1} \left[- \sum_{i=1}^{n-1} c_i e^{(i)} - f(\underline{x}, t) + \dot{x}_d^{(n)} \right] \quad (13)$$

$$u_{sw} = -g(\underline{x}, t)^{-1} \cdot \eta \text{sgn}(s) \quad (14)$$

But since f and g are still unknown, it is difficult to apply the control law (9) to an unknown nonlinear system. Moreover, the switching type condition u_{sw} will create chattering.

7. Synchronization Using SMC

Using the SMC technique, the problem of the controller design is divided into two independent steps:

Step 1: Selecting a sliding surface for desirable sliding movement

Step 2: Designing a controller to move any trajectory within the state space toward the sliding surface, where a non-continuous controller can be used to maintain the ultimate state on this surface. A special characteristic of this control mechanism is that while it is in the sliding mode, the system is robust with respect to parameter uncertainty and outer disturbances.

For this purpose, first a sliding surface must be selected; *i.e.* a time-varying surface $s(t)$ defined as

$$s = \sum_{j=1}^{\rho-1} k_j e_j + e_\rho \quad (15)$$

where e_j is the state error and ρ is relative degree, and the positive constants k are chosen so that the following polynomial becomes a Hurwitz polynomial:

$$\lambda^{\rho-1} + k_{\rho-1} \lambda^{\rho-2} + \dots + k_1 \quad (16)$$

This condition ensures that the movements will be limited to the surface $s=0$ and, therefore, the tracking errors $e=x-x_d$ (where x_d represents the desirable states) will converge to zero. Sliding mode or sliding state refers to the behavior of the system on the sliding surface.

A feedback control law u is selected in a way that satisfies the sliding condition. However, in order to account for uncertainties in modeling and disturbances, the control law must be non-continuous on $s(t)$. Since the switching control law may not be implemented perfectly (for example, the switching does not occur instantly and s is not known with infinite accuracy), this results in chattering which is not desirable in practical applications because it increases control activities and may also excite high-frequency dynamics (*e.g.* unmodeled structural modes, neglected delays, etc) which have not been considered during modeling.

We show the sliding mode dynamics as $\dot{s} = 0$.

By solving the equation above, an expression, known as equivalent control u_{eq} , is found for u . It can be thought of as a continuous control law which satisfies $\dot{s} = 0$ if the dynamics are known.

As noted in the previous sections, the equations for SA and AV are in the following forms:

$$\begin{aligned}
 SA : \ddot{x}_d &= -x_d + c_1(1-x_d^2)\dot{x}_d \\
 AV : \ddot{x} &= -x + c_2(1-x^2)\dot{x} + R(\dot{x}_d - \dot{x}) + u
 \end{aligned}
 \tag{17}$$

where c_2 is assumed to contain uncertainty. The objective in the synchronization problem is to make AV oscillator track SA oscillator. Therefore, for the system above to track $x(t) \equiv x_d(t)$, sliding surface is defined as

$$\begin{aligned}
 S &= \left(\frac{d}{dt} + \lambda\right)^{(\rho-1)} \tilde{x} = \dot{\tilde{x}} + \lambda\tilde{x} \\
 \tilde{x} &= x - x_d
 \end{aligned}
 \tag{18}$$

where $s(t)$ is both a dynamic and a locus.

$$\begin{aligned}
 \dot{S} &= \dot{\tilde{x}} + \lambda\dot{\tilde{x}} = \ddot{x} - \ddot{x}_d + \lambda\dot{\tilde{x}} \\
 &= -x + c_2(1-x^2)\dot{x} + R\dot{\tilde{x}}_d + u - \ddot{x}_d + \lambda\dot{\tilde{x}} = 0
 \end{aligned}
 \tag{19}$$

Then we have:

$$u_{eq} = x - c_2(1-x^2)\dot{x} - R\dot{x}_d + \ddot{x}_d - \lambda\dot{\tilde{x}}
 \tag{20}$$

Thus, equivalent control is obtained. The above value for u cannot be calculated since c_2 is unknown. Suppose that $\hat{c}_2 = c_2 + \Delta c_2$, where Δc_2 represents parameter uncertainty. Hence, for u we have:

$$\begin{aligned}
 \hat{u}_{eq} &= x - \hat{c}_2(1-x^2)\dot{x} - R\dot{x}_d + \ddot{x}_d - \lambda\dot{\tilde{x}} \\
 \Rightarrow \dot{S} &= (c_2 - \hat{c}_2)(1-x^2)\dot{x} \neq 0
 \end{aligned}
 \tag{21}$$

In order for the sliding condition to hold, despite of the uncertainties in the system dynamics, a discontinuous term is added to \hat{u}_{eq} on the surface $s=0$:

$$\begin{aligned}
 u &= \hat{u}_{eq} - k \operatorname{sgn}(S) \\
 \Rightarrow \dot{S} &= (c_2 - \hat{c}_2)(1-x^2)\dot{x} - k \operatorname{sgn}(S)
 \end{aligned}
 \tag{22}$$

where sgn is the sign function, and k is defined as follows in a way that the sliding conditions are met and sliding mode movement occurs.

$$k = (c_2 - \hat{c}_2)(1-x^2)\dot{x} + \eta, \quad \eta > 0
 \tag{23}$$

As seen here, discontinuity of k on the surface $s=0$ increases with the parameter uncertainty. By choosing a proper value for k , it is possible to reduce chattering as well as the time needed for attaining sliding mode [52]. This ensures that s will converge to zero within a finite time and will remain at this value for all subsequent times, and therefore, the errors e_1 and e_2 will asymptotically approach zero. However, this technique may lead to excitation of unmodeled high-frequency dynamics or improper switching within the system, thereby deteriorating the system performance or even causing instability. Researchers, therefore, always tried to address this issue. For this purpose, different methods have been employed to reduce or eliminate this type of excitation. One common technique used to resolve this problem is to replace discontinuous control with a continuous approximation. In this way, the discontinuous function $\operatorname{sgn}(s)$ is replaced with its continuous approximation $\operatorname{sat}(s)$. This technique works well in resolving the problem but often at the expense of a non-zero steady-state error. Here, we propose an appropriate method for synchronizing two oscillators with parameter uncertainty and even with different structures.

7-1 Stability

To observe dynamic stability of error in the proposed controller, the Lyapunov candidate function $V=s^2/2$ was chosen. The derivative of V is given by

$$\begin{aligned}
 \dot{V} &= \dot{S}S = [(c_2 - \hat{c}_2)(1-x^2)\dot{x} - k \operatorname{sgn}(S)]S \\
 \Rightarrow \dot{V} &= [(c_2 - \hat{c}_2)(1-x^2)\dot{x} - |(c_2 - \hat{c}_2)(1-x^2)\dot{x}|] |S| - \eta |S| < 0
 \end{aligned}
 \tag{24}$$

Note that $sgn(s)$ is always positive as long as $k > 0$ and $e \neq 0$.

Since V is a positive and decrescent function and \dot{V} is negative definite, the equilibrium point $(e_i = 0, i = 1, \dots, n)$ is uniformly stable; that is, $e_i(t) \in L_\infty$. It can be easily shown that $e_i(t), i = 1, \dots, n$ are square-integrable with respect to t , i.e. $e_i \in L_2$. According to Barbalat's Lemma [53], and for any initial conditions, we have $\dot{e}_i(t) \in L_\infty$, which means $e_i(t) \rightarrow 0$ as $t \rightarrow \infty$. Thus, S asymptotically converges to zero. Therefore, $x \rightarrow x_d$ when $t \rightarrow \infty$. This shows that the SMC has synchronized the two systems.

7-2 Simulation Results

To demonstrate the capabilities of the proposed method, in this section synchronization is carried out for two sample systems. The descriptive equations for the SA and AV oscillators are:

$$\begin{aligned}
 SA: & \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -w_1^2 x_1 + c_1(1-x_1^2)x_2 \end{cases} \\
 AV: & \begin{cases} \dot{x}_3 = x_4 \\ \dot{x}_4 = -w_2^2 x_3 + c_2(1-x_3^2)x_4 + R(x_2 - x_4) + u \end{cases}
 \end{aligned} \tag{25}$$

The initial conditions for the master and slave are (1,4) and (0.7, 2), respectively. Based on the physiological facts, a one-way coupling is considered here. The frequency is 60 pulses per minute for the first oscillator and 40 pulses per minute for the second oscillator. It is assumed that c_2 contains uncertainty and $\hat{c}_2 = c_2 + \Delta c_2$ where Δc_2 represents uncertainty. SA and AV oscillators, synchronized by the proposed method despite parameter uncertainty, are shown in Fig. 8.

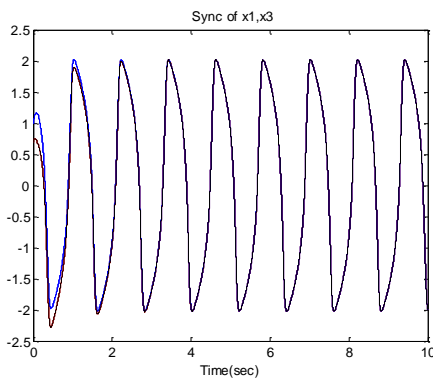


Figure 8. Synchronization using SMC

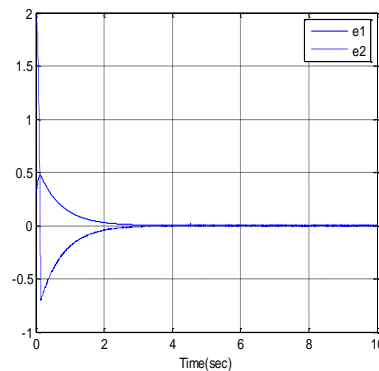


Figure 9. presents synchronization error which has diminished over time

8. PSO algorithm

PSO is a population-based stochastic optimization technique, which does not use the gradient of the problem being optimized, so it does not require being differentiable for the optimization problem as is necessary in classic optimization algorithms. Therefore, it can also be used in optimization problems that are partially irregular, time variable, and noisy. In PSO algorithm, each bird, referred to as a “particle”, represents a possible solution for the problem. Each particle moves through the D-dimensional problem space by updating its velocities with the best solution found by itself (cognitive behavior) and the best solution found by any particle in its neighborhood (social behavior). Particles move in a multidimensional search space, and each particle has a velocity and a position as follows:

$$v_i(k+1) = v_i(k) + \gamma_{1i}(P_i - x_i(k)) + \gamma_{2i}(G - x_i(k)) \tag{26}$$

$$x_i(k+1) = x_i(k) + v_i(k+1) \tag{27}$$

where i is the particle index, k is the discrete-time index, v_i is the velocity of i th particle, x_i is the position of i th particle, P_i is the best position found by i th particle (personal best), G is the best position found by swarm (global best), and $\gamma_{1,2}$ are random numbers in the interval $[0, 1]$ applied to i th particle. In our simulations, the following equation is used for velocity:

$$v_i(k+1) = \phi(k)v_i(k) + \beta_1[\gamma_{1i}(P_i - x_i(k))] + \beta_2[\gamma_{2i}(G_i - x_i(k))] \tag{28}$$

in which ϕ is inertia function and β_1, β_2 are acceleration constants. The flowchart of the standard PSO algorithm is depicted in Fig. 10.

9. Proposed synchronization schemes

As mentioned before, the synchronization scheme consists of two systems: the master and the slave (Fig. 11). In this scheme, an RBF- or “RBF + error integral”-based controller is used to make the states of the slave system follow the states of the master system, in the presence of uncertainties and external disturbances. It should be noted that in Fig. 11 $h(\cdot)$ can be any continuous function. In this section, two proposed methods of systems synchronization are described: (14) RBF-based nonlinear controller, (15) “RBF + error integral” model in which an integral term is added to RBF model to improve the robustness of the proposed controller. To optimize the parameters of these controllers, PSO as a continuous evolutionary algorithm is also used. The mathematical formulation:

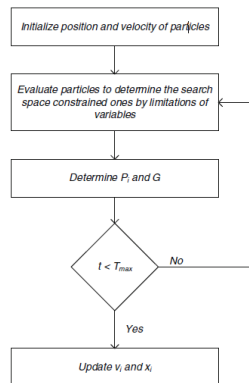


Figure 10. Flowchart of PSO algorithm

of the proposed methods is stated in the following subsections:

10. control by RBF model

Consider the control system in (15), for this system the following RBF-based controller is proposed:

$$u(t) = W^T \xi(e) \tag{29}$$

in which $u(t)$

is the control signal, and $e = [e_1, e_2, \dots, e_n]^T, e_i = x_{im} - x_{is}, i = 1, 2, \dots, n, x_{im}$ and $x_{is}, i = 1, 2, \dots, n$ are the states of master and slave systems, respectively. $W = [w_1, w_2, \dots, w_M]^T \in \mathfrak{R}^M$ is the weight matrix, and $\xi(e) = [\xi_1(e), \xi_2(e), \dots, \xi_M(e)]^T \in \mathfrak{R}^M$ is a set of basis functions of the corresponding RBF model. The basis function $\xi_i(e)^M$ of i th node in the hidden layer is considered a Gaussian function as:

$$\xi_i(e) = \exp\left(-\frac{\|e - c_i\|^2}{\delta_i^2}\right) \tag{30}$$

in which c_i and δ_i are the center and the width, respectively. Considering $c = [c_1, c_2, \dots, c_n]^T \in \mathfrak{R}^n$ and $\delta = [\delta_1, \delta_2, \dots, \delta_n]^T \in \mathfrak{R}^n$ the goal is to find the optimized $W = W^*, c = c^*$ and $\delta = \delta^*$ such that the following cost functional is minimized:

$$IAE = \int_0^T \|e(t)\| dt \tag{31}$$

Where $\|\cdot\|$ is the Euclidean norm of a vector. To find the optimized parameters $W=W^*, c=c^*$ and $\delta=\delta^*$, the PSO algorithm is used.

11. “RBF + error integral” model

As mentioned before, there may be modeling uncertainties and external disturbances in the control problem. Therefore, the controller should be robust enough such that it can cope with these uncertainties. Now, as a modification of the method proposed, the integral components are added to the basis function vector to increase the robustness of the system. Therefore, the following controller is proposed:

$$u(t) = W_I^T \xi_1(e) \tag{32} \text{ Where}$$

$$W_I = [w_1, w_2, \dots, w_M, w_{M+1}, w_{m+2}, \dots, w_{M+n}]^T \in \mathfrak{R}^{M+n}, \xi_1(e) = [\xi_{1I}(e), \xi_{2I}(e), \dots, \xi_{MI}(e), \int edt]^T \in \mathfrak{R}^{+n} \tag{33}$$

$$\xi_{iI}(e) = \xi_i(e), i = 1, 2, \dots, M \text{ and } \int edt = [\int e_1 dt, \int e_2 dt, \dots, \int e_n dt] \in \mathfrak{R}$$

The goal in this scheme is to find the optimized $W=W^*, c=c^*$ and $\delta=\delta^*$ such that the cost functional (31) is minimized.

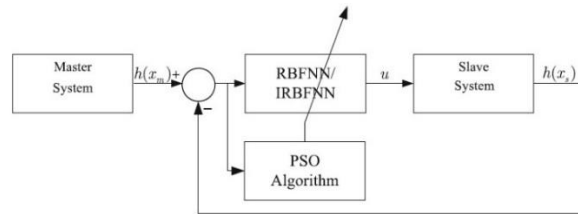


Figure 11. Block diagram of synchronization scheme

12. Simulation and experimental results

Figure 3 illustrates the block diagram of system. As mentioned before, the system consists of a master and a slave. Considering $h(\cdot)$ to be any continuous function, in system masking scheme, the message signal $m(t)$ is added to the output of the master system, $h(x_m)$. The controller is designed such that the master and the slave systems are synchronized. Thus by subtracting the output of the slave system, $h(x_s)$, from the resulted signal, the message signal can be thoroughly recovered. It should be noted that the controller should be designed such that it can cope with uncertainties and external disturbances.

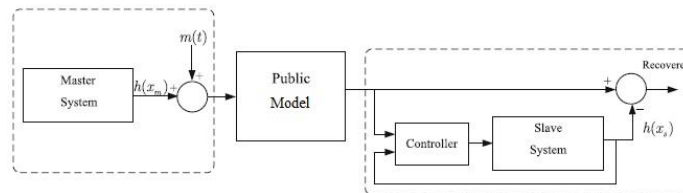


Figure 12. Block diagram of system masking scheme

The descriptive equations for the SA and AV oscillators are:

$$SA: \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -w_1^2 x_1 + c_1(1-x_1^2)x_2 \end{cases}$$

$$AV: \begin{cases} \dot{x}_3 = x_4 \\ \dot{x}_4 = -w_2^2 x_3 + c_2(1-x_3^2)x_4 + R(x_2 - x_4) + u \end{cases}$$

The initial conditions for the master and slave are (1,4) and (0.7, 2), respectively. Based on the physiological facts, a one-way coupling is considered here. The frequency is 60 pulses per minute for the first oscillator and 40

pulses per minute for the second oscillator. It is assumed that c_2 contains uncertainty and $\hat{c}_2 = c_2 + \Delta c_2$ where Δc_2 represents uncertainty.

SA and AV oscillators, synchronized by the proposed method despite parameter uncertainty, are shown in Fig. 14.

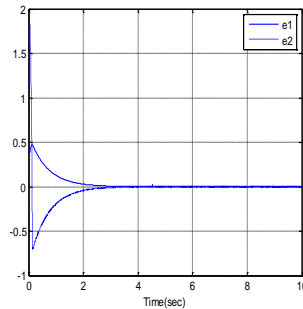


Figure 13. presents synchronization error which has diminished over time

Eventually, as seen in Fig. (14), the system output converges to the desirable output.

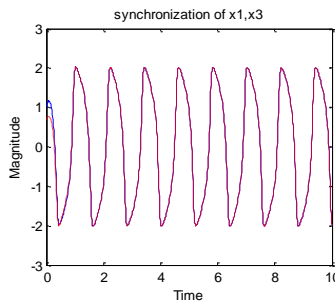


Figure 14. Synchronization using PSO optimized RBF-based controllers

As seen in this figure, the SA and AV oscillators have become synchronized once a time period is passed.

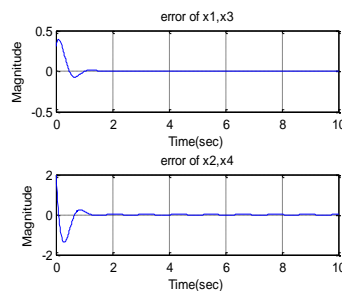


Figure 15. Synchronization error

The figure above presents the synchronization error. This error vanishes over time indicating that the two oscillators have become synchronized.

13. Two-way coupling

If two-way coupling is used for the two oscillators - which in physiological sense means that AV oscillator impacts SA oscillator as well, sometimes in a relatively weak manner – the equations become:

$$SA: \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -w_1^2 x_1 + c_1(1-x_1^2)x_2 + R_1(x_4 - x_2) \end{cases}$$

$$AV: \begin{cases} \dot{x}_3 = x_4 \\ \dot{x}_4 = -w_2^2 x_3 + c_2(1-x_3^2)x_4 + R_2(x_2 - x_4) + u \end{cases}$$

First, we select a value for R_1 which was about the one tenth of R_2 . The computation results indicated that two-way coupling has no effect on synchronization time. The value of R_1 was then increased, but no impact was observed on synchronization time. This is in line with physiology of heart as AV oscillator has negligible effect on SA oscillator.

CONCLUSION

The table below shows the results for the two methods. As seen from the table, PSO optimized RBF-based controllers outperforms in terms of synchronization time and variance of error.

SA and AV Node Oscillators	Synchronization Time(sec)	Error variance	Control effort(max)	Control effort (min)
PSO optimized	0.2	0.001152	149+	-149
RBF-based controllers				
SMC Control	3.7	0/0032	7.2	-31

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